

The physics of the centrality dependence of elliptic flow

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The centrality dependence of elliptic flow and how it is related to the physics of expansion of the system created in high energy nuclear collisions is discussed. Since in the hydro limit elliptic flow is proportional to the elliptic anisotropy of the overlapping region of the colliding nuclei, and in the low density limit to the product of the elliptic anisotropy and the multiplicity, we argue that the centrality dependence of elliptic flow should be a good indicator of the degree of equilibration reached in the reaction. Then we analyze experimental data obtained at AGS and SPS energies. The observed difference in the centrality dependence of elliptic flow could imply a change from hadronic to partonic degrees of freedom. Finally we exploit the multiplicity dependence of elliptic flow to make predictions for RHIC and LHC.

The goal of the ultrarelativistic nuclear collision program is the creation of the QGP – quark-gluon plasma – the state of deconfined quarks and gluons. It is understood that such a state requires (local) thermalization of the system brought about by many rescatterings per particle during the system evolution. It is not clear when and if such a dynamical thermalization can really occur. An understanding of these phenomena can be achieved by considering elliptic flow [1] recently studied at AGS [2] and SPS [3] energies. It will be shown how the strength of elliptic flow, v_2 , defined as the second coefficient in the Fourier decomposition of the particle azimuthal distribution [4], is a strong indicator of the degree of equilibration (thermalization) achieved in the system.

To discuss the centrality dependence of v_2 we start from the hypothesis that the system is *not* dense and its evolution can be described by the first correction to the collisionless limit [5]. Physically this means that the rescattering occurring during the system evolution changes the particle momenta very little on the average and the corresponding change in the distribution functions can be treated in first order as perturbations. This excludes the limit of dynamical thermalization which is an assumption of hydro models. We believe the low density limit is the case valid at AGS and SPS energies, possibly also at RHIC, but probably not at LHC (see discussion below). Under this assumption the final elliptic flow, v_2 , is proportional to the initial overlapping region elliptic anisotropy, ε , (introduced in flow analyses in [1]

and in its present form in [5,6]) and to the initial particle space density which defines the probability of particles to rescatter [5].

The initial geometry of the overlapping zone can be evaluated in a simple Glauber type model with a Woods-Saxon nuclear density. The results are weakly dependent on the weights used [7], such as the calculation of the number of wounded nucleons. What is important is that if one wants to compare different energies, e.g. AGS, SPS and RHIC, the nuclear geometry cancels out, and only the dependence on multiplicity is left. This is true provided that the “physics” of rescattering stays the same. If the physics changes then the scaling with multiplicity will be violated. This is a very important point if one reads it the other way around: if scaling is not observed then the physics has changed.

Under the assumption that the system is relatively dilute, the spectra distortion is directly proportional to the number of rescatterings, that is to the particle density in the transverse plane. In this limit the final elliptic flow (see a more detailed formula in [5])

$$v_2 \propto \varepsilon \frac{1}{S} \frac{dN}{dy}, \quad (1)$$

where $S = \pi R_x R_y$ is the area of the overlapping zone, with $R_x^2 \equiv \langle x^2 \rangle$ and $R_y^2 \equiv \langle y^2 \rangle$ being the initial geometrical sizes of the system in x and y directions, respectively. (The x - z axes lie in the reaction plane). The averages include a weighting with the number of collisions along the beam axis. The initial space elliptic anisotropy is defined as

$$\varepsilon = \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2}. \quad (2)$$

In our calculation we use a Woods-Saxon parameterization of the nuclear density with parameters $R_A = 1.12 \cdot A^{1/3}$, and $a = 0.547$ fm. More information on the use of different weights and the values of R_x^2 , R_y^2 , S and ε as a function of impact parameter can be found in [7]. The proportionality coefficient in Eq. (1) is defined by the “physics” of the rescattering. If the physics is the same in central and peripheral collisions then Eq. (1) yields the centrality dependence of v_2 .

As follows from Eq. (1) the elliptic flow increases with the particle density. Eventually it will saturate [8] at the

hydro limit, which would mean complete thermalization of the system. One can try to estimate the conditions needed for the hydro limit of flow development. In this regime the centrality dependence of elliptic flow is mainly determined by the initial elliptic anisotropy of the overlapping zone in the transverse plane [8], and the ratio of the two should be approximately constant as shown in the first such calculations done by Ollitrault [1]. From his results it follows that $(v_2/\varepsilon)_{hydro} \approx 0.27 - 0.35$, depending on the equation of state used (with or without QGP)¹. The calculations [9] give a somewhat smaller flow, resulting in $(v_2/\varepsilon)_{hydro} \approx 0.21 - 0.23$ (partly due to the realistic treatment of resonances which decrease the pion flow by about 15%).

Before discussing the experimental data we will first consider a realistic model. We take RQMD v2.3 [10] for our calculations. Fig. 1 top shows the comparison of the directly calculated v_2 of pions in Pb+Pb collisions at 158 GeV·A collisions at mid-rapidity ($-1 < y < 1$) with the expectation from the low density limit, v_2^{LDL} (Eq. (1) normalized to the same area under the curve). One can see rather good agreement, which indicates that no new physics enters as one scans the centrality from peripheral to central collisions (which is true in this version of RQMD). This also means that at all impact parameters RQMD is near the low density limit. The centrality dependence expected for the hydro limit is shown on the same plot by a dashed line also normalized to the same area under the curve ($v_2^{HYDRO} \approx 0.059\varepsilon$). Note the large difference between the two curves, which was not noted in [5]. Fig. 1 bottom shows that the ratio of v_2 to the expected functional form is flat for the low density limit but not for hydro.

A centrality dependence similar to the low density limit was also observed in [11] where a computer simulation of a pion gas expansion was studied. Now let us turn to the experimental data. At AGS energies the elliptic flow of charged particles and of transverse energy was measured by the E877 Collaboration. Unfortunately, the publication [2] containing the detailed pseudorapidity dependence for each centrality lacks a figure showing just the centrality dependence. Our estimates based on their data [2] of charged particle flow at

¹To avoid confusion, note the difference in definitions of ε used in this paper and α_x from [1]. For Pb+Pb collisions the maximal value of $\varepsilon \approx 0.44$ compared to $\alpha \approx 0.3$. Then, the results [1] yield $v_2^{\{p_t^2\}}/\varepsilon \approx 0.55 - 0.7$, where $v_2^{\{p_t^2\}}$ means the elliptic flow weighted with p_t^2 . Recent calculations [9] show that the particle elliptic flow is related to this quantity as $v_2 \approx 0.5 v_2^{\{p_t^2\}}$.

midrapidity are presented in Fig. 2.

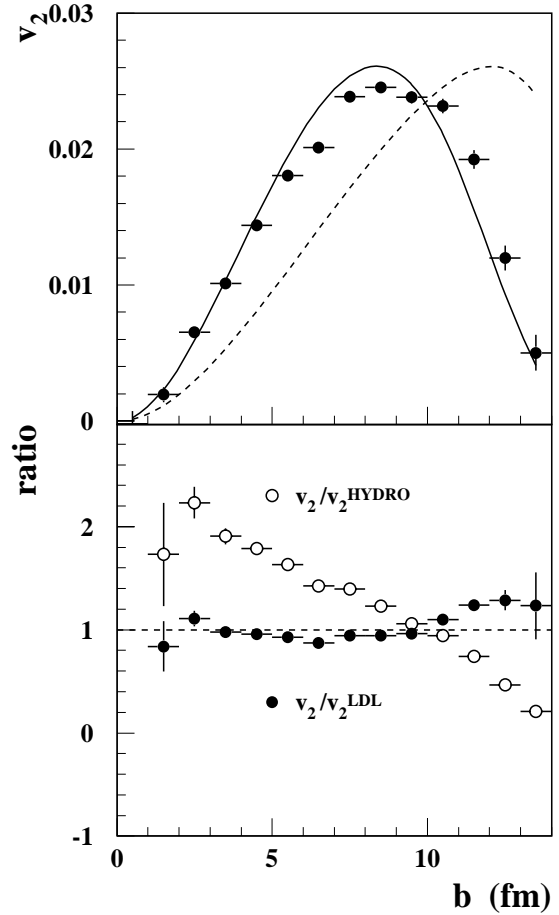


FIG. 1. Top: comparison of elliptic flow, v_2 , for pions from RQMD ver. 2.3 (filled circles) with the dependence expected for the low-density limit (solid line) and that expected for the hydro limit (dashed line). Bottom: ratios of v_2/v_2^{LDL} , and v_2/v_2^{HYDRO} .

The data indicate that at AGS the flow peaks at mid-centrality², consistent with the low density limit prediction and no change in physics with centrality. At SPS [13], preliminary data indicate that the elliptic flow peak moves towards peripheral collisions. This fact itself would hint at the hydro-dynamical picture of the system evolution. A more detailed look at the data shows that this is unlikely. First, the maximal value of elliptic flow ($v_2 \leq 0.04$) is significantly less than predicted by hydro calculations [1,9] (about 0.09–0.1). In [9] agreement was claimed between hydro and the NA49 mid-central

²A similar centrality dependence of transverse energy flow (from the same data [2]) can be found in the thesis of Chang [12].

data [3] leading to their conclusion of complete equilibration. However, this comparison was done for $p_t < 0.3$ GeV/c and it could be that the p_t dependence of v_2 in the hydro model does not agree with experiment. In the hydro limit elliptic flow should depend only on the initial space elliptic anisotropy, ε . The preliminary NA49 data indicate that the ratio v_2/ε , at least for semi-central collisions, is likely increasing with centrality [13]. This centrality dependence (natural for the low density limit) implies that we still could be far from the hydro regime.

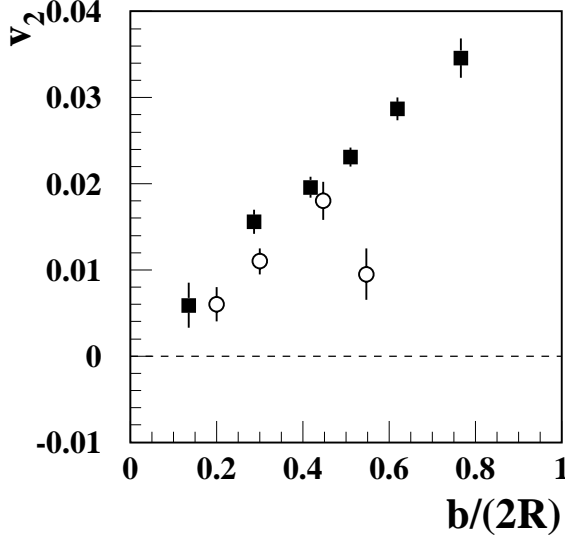


FIG. 2. Elliptic flow at the AGS (open circles) and the SPS (filled squares).

One can argue that, taking into account systematic uncertainties, the preliminary SPS data for v_2/ε are consistent with being constant as a function of centrality. In this case it would indeed mean that the system has equilibrated and the hydro regime has been reached. The low absolute strength of the elliptic flow in this case would indicate that the equilibration happens at a rather late time when the spatial anisotropy ε has decreased due to initial “free streaming”. We do not exclude this possibility but must wait for the final SPS data and the coming RHIC data to answer the question. In the mean time let us come back to the other interesting possibility, that at SPS the hydro limit is not reached.

Assuming that at SPS we are in the low density limit, the observed centrality dependence of elliptic flow would indicate that the physics of rescattering is different in central and peripheral collisions. A natural explanation for this would be that peripheral collisions are described by hadronic scattering while in central collisions partonic (re)scattering becomes important.

Summarizing, the following picture emerges: at AGS energies, the physics of rescattering which defines the sys-

tem evolution is hadronic in nature, while at SPS it is the same for peripheral collisions, but for central collisions the physics is likely to be partonic. The partonic picture will remain at RHIC energies, with some extension toward more peripheral collisions. At RHIC equilibration becomes more important, but it is not clear if complete thermalization will be reached. At LHC energies the parton densities could become so high that (partonic) rescattering would lead to dynamical equilibration of the (partonic) system (creation of regions of real QGP) and consequently to a hydro-dynamical type of system evolution.

The above picture implies that the shape of the centrality dependence of elliptic flow would change continuously with beam energy. At the AGS, the elliptic flow is peaked at an impact parameter value slightly higher than R_A , while at SPS energies the peak moves toward more peripheral collisions because the physics of central collisions could have changed from hadronic to partonic which leads to weaker flow than one would expect taking into account the increased multiplicity. If thermalization is not reached at RHIC, elliptic flow will peak once more at mid-central collisions because the physics of the peripheral and central collisions will be the same – partonic rescattering. At even higher energies at LHC, the elliptic flow should peak at more peripheral collisions just as predicted by hydrodynamic calculations.

The schematic view of the picture which emerges from these observations is presented in Fig. 3, where the ratio of elliptic flow to the initial space elliptic anisotropy is presented as a function of initial particle density.³ In this plot we use the experimental charged particle multiplicity, assuming that it is proportional to the total particle multiplicity and also to the initial particle multiplicity. For the experimental values we use dN_{ch}/dy at

³Many things shown in this plot have large uncertainties. The hydro limits can depend slightly on the initial particle density [1,9] and, more importantly, on the time of thermalization of the system. The values shown are an average of the results of [1,9]. The predictions for the case without QGP are only for the EoS of a massless pion gas. Resonances can soften the EoS and lead to weaker flow. The uncertainty in the experimental points is mainly in the determination of the collision centrality required for calculation of the initial space elliptic anisotropy and the area of the overlapping region. The data points correspond to the centrality determined from the fraction of the total cross section corresponding to each centrality bin. Higher centralities were estimated from experimental measurement of the number of participants [14]. Finally, the smooth dashed curves are just schematic illustrations for hadronic and partonic scenarios and the solid curve includes a phase transition.

mid-rapidity from [14,15].

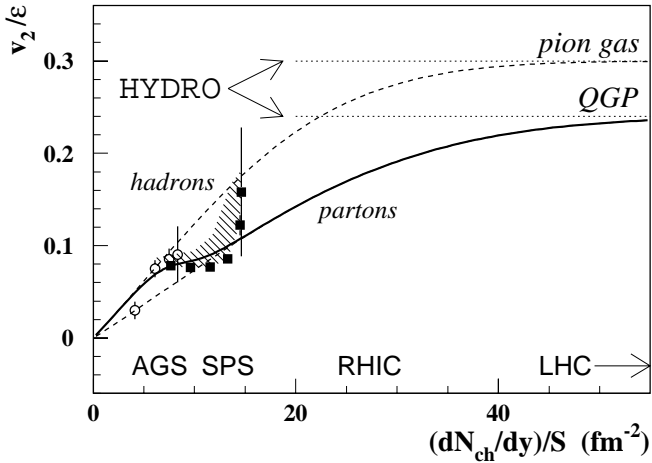


FIG. 3. Elliptic flow divided by the initial space ellipticity at the AGS (open circles) and the SPS (filled squares). The shaded area shows the uncertainty in the SPS experimental data due to the uncertainty in the centrality determination. See text and footnote for the description of the curves and hydro limits.

In the limit of very low density the objects which rescatter must be hadrons. At some critical density a partial deconfinement happens. Parton density becomes high enough such that the color charge can freely move in the perpendicular plane. Each parton is always close enough to other partons which screen its color⁴. Once the motion in the perpendicular plane becomes easier, the elliptic flow decreases. Note that the system still can be far from being dynamically thermalized, which would occur only at even higher particle densities. Even more important, which was not fully appreciated in [6,5], such a significant change only can happen if the system is not thermalized. See also the discussion of this question in [9] along with the discussion of the possibility of observation of the QGP to hadron gas phase transition.

To prove or disapprove the picture described above one needs more accurate data on the centrality dependence of elliptic flow. We would like to emphasize the importance of flow measurements not only at medium impact parameters but in the full range of centrality including rather central collisions where the anisotropic flow is small. The measurement of elliptic flow and its centrality dependence at RHIC thus becomes very important. Different models predict different rapidity densities for RHIC and LHC. Assuming that they are higher than at SPS by factors of 2 and 8, respectively, we have indicated the regions

⁴This picture is very close to the parton percolation model discussed by Satz [16] for J/Ψ suppression.

expected in Fig. 3. The new SPS data taken at 40 GeV-A energy are also of great interest since they should scan the “hadronic-to-partonic” transition region.

Note that our picture of nuclear collisions and QGP production is different from what is usually discussed, which assumes thermal equilibrium even at rather low beam energies, when QGP is not expected, and then with an increase in collision energy, formation of regions of QGP. We believe that what could happen is that the deconfinement can occur before dynamical thermalization is achieved [17] and that the centrality dependence of elliptic flow would be a good indicator of this. It would be interesting to investigate other consequences of this picture. For example, it should result in a specific dependence of HBT radii on particle transverse momenta [18].

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